A 0-1 Integer Programming Model For Exam Invigilators Assignment Problem

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***Abstract*—Examination invigilators assignment is an important operational problem that takes place in all academic institutions. Creating a good exam invigilator schedule that will satisfy lecturers, staff, and institution is a difficult and tedious task as there are numerous factors and constraints that need to be taken into consideration. This paper presents a 0-1 integer programming model for solving the exam invigilators assignment problem at Universiti Teknologi Mara (UiTM) Pahang, Raub campus. The objective of the model is to fairly assign both the invigilator and chief invigilator duties among the academic staff. The formulated model is sufficiently flexible to deal with many different operational rules and requirements found in most academic institutions; for example, lecturers do not invigilate their own subjects, an invigilator should not be scheduled to invigilate more than once in the same time slot, and not more than one chief invigilator is needed for an exam. The model was then tested using real data from UiTM Raub campus. The results from the experiment demonstrate that the proposed model can produce good solutions that satisfy all the constraints.**

***Keywords*—integer programming; invigilators assignment; examination timetabling.**

I. Introduction

Examination timetabling problem is a typical problem faced by most academic institutions across the world. In general, the problem involves assigning a set of examinations into a fixed number of time slots and rooms, so that no student is required to take more than one examination at any time [1]. Constructing a high quality examination timetable can be an extremely difficult task and its manual solution often requires a significant amount of time, sometimes several days or even weeks. The complexity of the problem depends largely on the number of students, number of courses taught, number of examination rooms, and the length of the examination period.

The examination timetabling problem is subject to a number of constraints that are usually divided into two categories: hard and soft constraints [2]. Hard constraints are constraints that must not be violated under any circumstances. Soft constraints, on the other hand, may be tolerable, but must be minimised as much as possible. The sets of hard and soft constraints differ significantly in all institutions depending on their particular needs and limited resources.

Exam invigilators assignment is one aspect of the examination timetabling problem. The problem concerns with assigning invigilators to examinations and rooms in such a way that there are no conflicts or clashes. This process is often done separately from the scheduling of examinations to time slots and rooms. The invigilator assignment problem has not been extensively investigated as much as the other examination timetabling problems yet. According to [3], this may be partly due to the fact that no datasets are available for the problem. Furthermore, in some institutions, it is not common to have invigilators during the exams and there are also universities where students can have their exams over web services [4]. Less attention is paid to this problem. A survey conducted by [5] found that twenty nine percent of British universities agree that the task of assigning invigilators is a major problem. Studies by [6] and [7] also reported that many invigilators are not satisfied with their individual schedule. The findings of these studies show that it is essential to have a mathematical model that could solve the problem in a systematic way so that the assignment produced does satisfy all the requirements and constraints.

In this paper, a mathematical programming model based on 0-1 integer programming is presented to solve the exam invigilators assignment problem at Universiti Teknologi Mara (UiTM) Pahang, Raub campus. The model aims to satisfy a number of constraints, for instance, staff do not invigilate their own subjects, an invigilator should not be scheduled to invigilate more than once in the same time slot, and not more than one chief invigilator is needed for an exam. Furthermore, this approach could be applied to any other educational institutions that encounter this type of problem.

The remaining parts of this paper are organised as follows. In the next section, related works on the invigilator scheduling problem are presented. Section 3 presents the description of the invigilator scheduling problem at UiTM Pahang, Raub campus, including the assumptions of the model. The 0-1 integer programming formulation of the problem is discussed in Section 4, which comprises a complete description of the hard and soft constraints as well as the objective function of the model. The solution provided from the proposed model is presented in Section 5. Finally, the conclusion and possible future works are given in Section 6.

II. Literature Review

The exam invigilators assignment problem varies among institutions, depending on their particular needs and resources. Nevertheless, there are some common requirements that will be the basis of the general system. A lecturer preference survey conducted by [6] suggests that invigilators prefer to have between 2-3 invigilation duties with one or two days gap, and lecturers with other responsibilities such as administrative or research works should be given a reduced number of invigilation duties. The study also presents several hard constraints that should be fulfilled to solve the invigilators assignment problem. Among the constraints are:

- invigilators must not be scheduled to more than one room in a time slot.
- invigilators cannot invigilate their own exam papers.
- invigilators must be assigned fairly.

Over the past several decades, a significant number of articles have been published in the literature and a wide variety of approaches have been proposed to tackle the timetabling problems. Reference [8] classified these approaches into four broad categories: sequential techniques, cluster techniques, constraint-based techniques, and metaheuristics. Reference [9] later added three more categories namely, multicriteria techniques, case-based reasoning techniques, and hyperheuristics or self adaptive techniques. An extensive survey on the literature concerning examination timetabling problems can be found in [10] and [11].

Several researchers have employed a genetic algorithm to solve the exam invigilators assignment problem. Reference [12] for instance used the genetic algorithm to find a solution that does not have overlaps in exams or invigilation duties, while invigilator preferences are satisfied as much as possible. In [13], a basic genetic algorithm framework is combined with a simple user interface based on readily available software tools to develop a computer-based system for assigning invigilators.

Another computer-based system for exam invigilators assignment has been developed by [7]. The system that optimises lecturer preferences enables the lecturers to view the examination timetable, choose their preferred invigilation time slots, specify the examination date and time of their own subjects, and view their individual schedule. On top of that, the system also allows lecturers to provide feedback and any other relevant information to the invigilation scheduling committee.

A common approach to solve the invigilators assignment problem is to mathematically formulate the problem using mathematical programming. Among the studies using this approach are those of [3] and [4]. Reference [3] formulated a mathematical programming model based on integer programming for Universiti Malaysia Pahang (UMP). The model considers three other hard constraints in addition to the ones presented in [6]. The constraints are as such: chief invigilator must be a lecturer, all staff must invigilate a maximum of three examinations within the exam period, and the total number of invigilators assigned to each room has to equal the number of invigilators required for each room. Apart from that, they also proposed a constructive algorithm that can produce good quality solutions compared with the UMP software. Using multiobjective mixed integer programming, [4] developed a web-based automated system for solving the invigilators assignment problem. The system optimises objectives related to assignment cost, total assignment on individual loads, and total assignment on undesired time slots. The system is tested on real data provided by Industrial Engineering department of Eskisehir Osamngazi University.

Reference [14] formulated the problem of assigning invigilators to exams as a multiobjective integer programme with a weighted objective function that combines a preference function and a workload-fairness function. It uses the concept of combining good solutions in order to yield a better solution. To solve the formulated model, they used a solution technique based on scatter search. A more recent research by [15] formulated an invigilator-exam timetabling problem using the non-preemptive goal programming approach. The model offers more fairness by incorporating several preferences related to the equity of the number of invigilating tasks. Furthermore, the proposed model has flexibility in terms of embracing new rules and/or criteria.

III. Invigilators Assignment at UiTM Raub Campus

UiTM Raub campus is one of the UiTM campuses with the total number of students approaching 3,000. The university has three faculties offering five diploma programmes in the fields of Business, Public Administration, Computer Science, and Statistics. At the end of each semester, these students must sit for examinations for a couple of weeks. The examination timetable is prepared by the Examination Unit at the UiTM main campus. Once the examination timetable is available, it is sent to all faculties and branch campuses for rooms and invigilators assignment. Currently, the invigilator assignment for UiTM Raub campus is prepared manually by the Academic Affairs Division. The process requires several days of work and the solution obtained sometimes fails to adhere to some requirements imposed by the scheduler. For these reasons, a mathematical programming model has been formulated for the problem based on the following assumptions:

1. The examination timetable is readily available.
2. The examination rooms with sufficient capacity are available.
3. The model focuses on scheduling academic staff for the exams.
4. Each staff has an equal chance to be selected as a chief invigilator.

IV. Problem Formulation

In this section, the 0-1 integer programming model for the exam invigilators assignment problem is described in detail. Reference [3] constituted the core of the model, although there are a few differences stemming from specific rules in the following lists contain all sets, indices, parameters, and decision variables.

Sets:

N Set of all examinations

S Set of academic staff
R Set of rooms
T Set of time slots

Indices:

i Index for exams, $i \in \{1, \ldots, N\}$
s Index for staff, $s \in \{1, \ldots, S\}$
r Index for rooms, $r \in \{1, \ldots, R\}$
t Index for time slots, $t \in \{1, \ldots, T\}$

Parameters:

S_r The number of invigilators required in each room r.
a_{is} The exam-staff matrix. 1 denotes the lecturer teaches the course in that semester, 0 otherwise.
v_{it} 1 if examination i is scheduled on time slot t, 0 otherwise.
w_{ir} 1 if examination i is assigned to room r, 0 otherwise.
z_{rt} 1 if room r is assigned to time slot t, 0 otherwise.

Decision Variables:

x_{srt} 1 if staff s is assigned to invigilate in room r in time slot t as the chief invigilator, and 0 otherwise.
y_{srt} 1 if staff s is assigned to invigilate in room r in time slot t as an invigilator, and 0 otherwise.

Objective function:

The objective function of the model is constructed with the aim to find a schedule that keeps the number of invigilation duties among the staff balanced. It can be formulated as [3]:

$$\min \sum_{s=1}^{S} f\left(x_{srt}, y_{srt}\right) \qquad (1)$$

where

$$f(x_{srt}, y_{srt}) = \begin{cases} 0 & if \ \sum_{t=1}^{T}\sum_{r=1}^{R}(x_{srt}, y_{srt}) \le \frac{\sum_{t=1}^{T}\sum_{r=1}^{R} z_{rt} S_r}{S} \\ 1 & otherwise \end{cases}$$

The objective function (1) can be rewritten as

$$\min \sum_{s=1}^{S} u_s \qquad (2)$$

subject to

$$\sum_{t=1}^{T}\sum_{r=1}^{R}\left(x_{srt} + y_{srt}\right) - M u_s \le \frac{\sum_{t=1}^{T}\sum_{r=1}^{R} z_{rt} S_r}{S} \qquad (3)$$

where M is a large positive number and u_s ($s = 1,2,\ldots S$) are indicator variables restricted to be either zero or one.

Constraints:

The constraints for the exam invigilators assignment model are briefly listed as follows:

a) Invigilators or chief invigilators cannot invigilate their own exam paper.

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{r=1}^{R}\left(a_{is} v_{it} w_{ir}\right)\left(x_{srt} + y_{srt}\right) = 0 \ \text{ for all } s \in S \qquad (4)$$

b) No lecturer is assigned to multiple rooms at the same time.

$$\sum_{r=1}^{R}\left(x_{srt} + y_{srt}\right) \le 1 \ \text{ for all } s \in S \text{ and for all } t \in T \qquad (5)$$

c) All lecturers are required to invigilate exams not more than k times within the exam period.

$$\sum_{t=1}^{T}\sum_{r=1}^{R}\left(x_{srt} + y_{srt}\right) \le k \ \text{ for all } s \in S \qquad (6)$$

d) Each staff is assigned as a chief invigilator not more than l times.

$$\sum_{t=1}^{T}\sum_{r=1}^{R} y_{srt} \le l \ \text{ for all } s \in S \qquad (7)$$

e) The total number of staff to invigilate room r in time slot t must be equal to the number of invigilators required for each room.

$$\sum_{s=1}^{S}\left(x_{srt} + y_{srt}\right) = z_{rt} S_r \ \text{ for all } r \in R \text{ and for all } t \in T \qquad (8)$$

f) There is only one chief invigilator for room r in time slot t.

$$\sum_{s=1}^{S} y_{srt} = z_{rt} \ \text{ for all } r \in R \text{ and for all } t \in T \qquad (9)$$

V. Model Implementation

This section presents the implementation of the model into a case of exam invigilator scheduling problem at Universiti Teknologi Mara (UiTM) Raub campus. The problem (for March 2016) consists of planning 87 different examinations for 19 days using 8 rooms of different capacities. There are two time slots reserved for examinations per day. The university has 105 academic staff who can serve as invigilators during the exams.

The optimal solution is obtained in a few seconds by using MATLAB R2014a software programme. The results are summarised in Table I and Table II. Based on the result in Table I, it is clear that the proposed exam invigilators assignment model could produce a solution that satisfies all the constraints without any cost to the objective function. Table II shows the results of the invigilators assignments using the formulated model, which constitutes the main output of the model. The invigilators' duties for each exam are provided in the fourth and eighth columns of Table II. The total number of invigilation duties for each staff is obtained as either 1 or 2.

TABLE I. INVIGILATOR SCHEDULING RESULT

Constraints	Proposed approach
Invigilators or chief invigilators cannot invigilate their own exam paper.	Yes
No lecturer is assigned to multiple rooms at a same time.	Yes
All lecturers are required to invigilate exams not more than 2 times within the exam period.	Yes
Each staff is assigned as a chief invigilator not more than one.	Yes
Each room should be assigned the required number of invigilators.	Yes
Each room should be assigned only one chief invigilator.	Yes
Objective Value	0

TABLE II. THE INVIGILATORS TIMETABLE

Day	Timeslot	Room	Invigilator	Day	Timeslot	Room	Invigilator
1	1	1	5, 29, 30, 38 (C), 39, 78, 99	9	16	1	18, 37, 50, 59 (C), 72, 101
		2	85 (C)			2	97 (C)
		3	88, 101 (C)			3	63 (C)
	2	1	7, 22, 41, 63, 93 (C), 94, 104			4	25 (C)
		2	31 (C), 66		17	1	77, 83 (C)
		3	51, 67 (C)			3	86 (C)
		4	56 (C)	10	18	1	11, 61, 69 (C), 71
		5	90 (C)		19	1	30, 41, 47 (C), 52
		6	75 (C)	11	20	1	20 (C), 83, 93
		7	79 (C)		21	1	9, 12, 28, 40, 76 (C), 85
		8	100 (C)			3	60 (C)
2	3	1	20, 33 (C), 43, 44, 47, 95	12	22	1	12 (C), 14
		2	13 (C), 55			3	22 (C)
		3	102 (C)		23	1	6 (C), 43, 48, 68, 101
	4	1	4, 5, 46, 73, 78, 105 (C)	13	24	1	2 (C), 49, 71
		2	15 (C)		25	1	15, 54, 62 (C), 84, 87
3	5	1	7 (C), 61, 66, 79, 82, 86	14	26	1	36, 57, 70 (C)
		2	11 (C)		27	1	10, 39 (C), 76, 98
	6	1	29, 45, 46, 92 (C)	15	28	1	19, 35, 36 (C), 51, 74
4	7	1	1, 18, 27, 28 (C), 37		29	1	77 (C), 80
	8	1	23 (C), 24, 49, 81, 94, 95	16	30	1	16 (C), 21, 55, 64, 69
		2	96 (C)		31	1	10, 35, 57 (C), 58, 98
5	9	1	2, 3, 53 (C), 65, 73, 92	17	32	1	8, 26 (C), 33, 60, 103
	10	1	17 (C), 32, 34, 53, 54		33	1	6, 45, 65, 89 (C)
6	11	1	48 (C), 56, 72, 105	18	34	1	21, 84 (C), 91, 96, 97, 103
	12	1	8, 38, 58, 70, 99 (C)		35	1	32 (C), 44, 87
		2	74 (C)	19	36	1	50 (C), 62, 100, 104
		3	42 (C)		37	1	16, 27 (C), 80, 90
7	13	1	14 (C), 19, 81				
8	14	1	4 (C), 9, 64, 75, 82				
	15	1	24, 52, 88 (C)				

C = Chief Invigilator

VI. CONCLUSION

The process of assigning invigilators to exams is a challenging task. Besides, it is always difficult to reach the optimum assignment by solving the problem manually. In this paper, a mathematical model for the exam invigilators assignment problem is presented. The model that is formulated using the 0-1 integer programming approach includes several common hard constraints that have to be fulfilled. This model has been successfully tested on real data from UiTM Raub campus. The results demonstrate that the model can produce a good solution that adheres to all the requirements.

Concerning the constraints of the model, there are several practical requirements that can be included in the future, e.g. the number of invigilators must be proportional to the number of students, academic staff with administrative work should be given a reduced number of invigilation duties, and the chief invigilator in a large room should be an experienced lecturer. The addition of these constraints, however, may create certain difficulties in terms of attaining an exact optimum solution. Thus, further research into the use of heuristic or metaheuristics algorithms to solve this problem is needed.

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